Do Representations Help College Students Solve Mathematical Word Problems?

Brian D. Beitzel, Richard K. Staley, and Nelson DuBois
SUNY Oneonta

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Abstract: We investigated the use of two different kinds of mathematical representations in helping college students solve probability word problems. Two types of multiple-event probability problems were presented: joint probability of independent events (with replacement) and total probability of non-mutually-exclusive events. The specific-representations group was instructed in how to use tree and Venn diagrams; the matrix group was trained to use a matrix to solve both types of probability problems. The control group was instructed only in the formulation of equations. Results indicated that the control group outperformed the specific-representations group on the transfer problems; there was no difference between the matrix group and the other two groups on transfer problems. Results are discussed in terms of cognitive load.

Purpose

The major purpose of our investigation was to examine the influence of different types of representation on solving mathematical word problems.

Theoretical Framework

Why do many students have so much difficulty solving word problems? Although many factors are involved, there is general agreement that solving word problems requires four interrelated activities (Fuson, Hudson, & Pilar, 1997; Reed, 1999). First, given a word problem, the student reads the problem to understand the non-mathematical aspects of the context described in the problem (the *situation* conception). Second, while reading the problem and perhaps re-reading the problem, the student attempts to understand the mathematical situation presented in the text (the *mathematized* conception). These two processes are interrelated because the student cannot fully understand the mathematical situation until the semantics of the problem have been processed (Hegarty, Mayer, & Monk, 1995; Marshall, 1995; Nathan, Kintsch, & Young, 1992). Third, the student then needs to plan a solution method and generate the appropriate mathematical expressions (the *solution* conception). Finally, given the mathematical expression or expressions, the student computes the solution.

If the problem solver correctly identifies the underlying mathematical schema and knows how to solve the problem, these processes require virtually no space in working memory. In contrast, if the solution conception is not readily apparent to the student, the demand on working memory may exceed working memory capacity, thus making the problem very difficult (Cooper & Sweller, 1987). It should be noted that often the first two problem-solving phases present more difficulty for the student than the actual computation (Cardelle-Elawar, 1992; Robitaille & Garden, 1989; Stern, 1993).

Thus a critical question in need of much further investigation is how to help students become more successful in the earlier problem solving phases by employing strategies to manage the large amounts of complex information in problems. Current research indicates that students need strategies to convert information in a problem into coherent external representations (Hegarty & Kozhevnikov, 1999; Novick, 1990; 2003; Novick & Hmelo, 1994) and then use those representations to solve the problem. Many lines of research support the role of representations in problem solving.

Not only do representations increase the likelihood that students will acquire schematic knowledge, but they also reduce the load on working memory associated with solving complex problems. For example, Novick (1990) suggests that representations are *central* to the problem solving process. Further, even when the underlying structural features of two problems are sufficiently different to preclude the use of the same solution procedure, similarities with the representations at a more general level of description may facilitate the transfer of that representational knowledge to new situations (Novick, 2003). Although representational studies in the past have been conducted with specific problem types (Marshall, 1995; Willis & Fuson, 1988), little research has been conducted on representational transfer effects, particularly in relation to the provided-example paradigm. Does teaching students how to employ systematic representational strategies by providing the students with worked examples demonstrating the strategies increase mathematical performance through the process of representational transfer (Novick, 2003)? And which representational structures might work best for problem solving?

In a recent study, for example, Gerjets, Scheiter, & Catrambone (2006) contrasted a molar-example format (emphasizing task features and application of formulas) with a modular-example format (emphasizing smaller meaningful groups of solution steps that can be understood in isolation). The modular approach treatment led to better problem solving performance, learning time, and cognitive load regardless of the number of problem categories taught, the type of learning task, and the learners' level of domain-specific prior knowledge. In the present study we employed a matrix treatment, following Gerjets et al.'s modular approach, because the inherent structure of the matrix should afford a much wider range of applications than more specific types of representations, such as the Venn diagram or the tree diagram. Additionally, the matrix representational structure guides the students' selective attention by breaking down the complex solutions into smaller steps, thus reducing the intrinsic cognitive load associated with complex problems. Further, because of the coherent structure of the matrix, students should be able to readily compare and contrast each component part of the problem while visually inspecting the relationships amongst the entities in the problem and their associated values.

To investigate this issue, we designed two representational treatments contrasted with a typical procedural treatment. One of the representational treatments required use of specific concrete spatial representations (tree and Venn diagrams); the other representational treatment introduced a more general, less specific representation (a matrix) that can be employed in a variety of situations. For the alternate non-representational treatment condition, students were instructed how to solve problems by generating appropriate mathematical expressions. This condition, of course, bypassed any reference to representation construction.

It was hypothesized that both of the representational treatment groups would outperform the procedural group on both near and far transfer problems, and that the matrix representation group would outperform the tree/Venn diagram group on the far transfer tasks.

Method

Participants

The participants were 81 undergraduate students at an Eastern college enrolled in multiple sections of an introductory educational psychology course. They participated in the study for extra course credit.

Materials

Instructional material was developed explaining how to solve two types of multiple-event probability problems: (a) joint probability of independent events (with replacement) and (b) total probability of non-mutually exclusive events. The materials deployed the worked example procedure with fading that was developed by Renkl, Atkinson, and Maier (2000). The instructional material consisted of a tutorial on basic probability concepts, an exposition of how to solve problems of each of the two problem types, and eight practice problems – four word problems for each of the two types of probability problems. The instructional materials were identical across all treatments, except that the matrix treatment included matrix representations of the problems (with instructions on how to construct a matrix); and the specific-representations treatment included tree diagram representations for the joint probability of independent events problems and Venn diagrams for the total probability of non-mutually exclusive events problems. All instructional materials verbally described how to solve each type of problem in a series of steps that involved (a) constructing an appropriate representation (for the two representation treatments only); (b) determining the number of favorable outcomes and total possible outcomes for each event, (c) solving for the desired probability.

The dependent measure was the number of correct solutions to 16 word problems representing near transfer (8 problems) and far transfer (8 problems). The 8 near-transfer problems consisted of 4 of each type of problem presented in the instructional materials and differed from the latter in terms of surface features or story line. The 8 far-transfer problems were different in both surface and deep structure from the practice problems; however, the skills acquired during the instructional phase were sufficient to solve both the near- and far-transfer problems. For example, two of these far-transfer problems were combination problems requiring application of both joint probability and total probability equations. Participants were asked to show all of their work, but were not required to use any specific approach to solving the problems.

A questionnaire was also developed to gather information about participants' college-level mathematical background and proficiency.

Design

The experimental design was a 3 (specific representations vs. matrix representation vs. procedural) × 2 (near-transfer problems vs. far-transfer problems) with repeated measures on the last factor. Both pretest knowledge of probability concepts and time to complete the instructional treatment were used as covariates. All participants were randomly assigned to one of the three instructional treatment groups.

Procedure

Participants were tested in groups but completed their packets independently. The packets were structured such that the experiment was divided into three phases. In the first phase students were asked to complete a questionnaire about their educational history, including all math courses they had taken in college, their grade, and how long ago they had taken the course. Next they completed a pretest of probability concepts; this pretest presented nine word problems that required basic knowledge of probability to successfully solve. The second phase consisted of studying the instructional material with worked examples that represented the treatment to which each participant was assigned. In the third phase participants completed the 16 problems that served as the dependent measure for the study. In each phase students were instructed to write the times at which they started and completed that phase so that the number of minutes spent could be calculated.

Results

As stated previously the experimental design was a 3 × 2 design with 3 levels of the between-subjects factor (procedural vs. specific representations vs. matrix representation) and two levels of the within-subjects factor (near-transfer vs. far-transfer). A Fisher LSD approach (Levin, Serlin, & Seaman, 1994) was used to analyze the data. An analysis of covariance was conducted, with prior knowledge of probability concepts (represented by pretest scores) and study time (number of minutes spent completing the dependent measure) entered as covariates. There was a main effect for condition, $F(2, 67) = 4.75$, $p = .01$, but no interaction or main effect for near vs. far transfer (both *p*s > .05).

Pairwise comparisons demonstrated that the procedural group (M = 4.80, SE = 0.38) outperformed the specific-representation group (M = 3.15, SE = 0.36) in the number of word problems that they were able to solve, $p < .01$. There was no difference between the specific-representation group and the matrix group (M = 4.05, SE = 0.34) or the procedural and matrix groups, all *p*s > .05.

Conclusions/Implications

In this study the specific representations appeared to hinder learning how to solve probability problems, at least as compared to the procedural treatment without any representations. Students given no representations solved more problems than students given the specific representations (i.e., tree and Venn diagrams). Descriptively, the advantage appears to be on near-transfer problems—problems similar to those encountered during the instruction. Additionally, the specific representations, which are much like those most often found in textbooks, had a less beneficial effect on problem-solving than the procedural treatment. The problem-solving performance of the more abstract matrix, or tabular, representation treatment was not significantly different from that of the procedural treatment, or from the specific-representations treatment.

The use of representations to help students understand mathematical word problems—a common instructional practice—may not always be beneficial. The results of this study suggest that at least some types of representations (e.g., tree and Venn diagrams) may not facilitate college students' solution of probability problems.

Participants in the specific-representations condition used either tree or Venn diagrams on 47% of the problems (M = 7.56, SD = 3.16) to help them solve the transfer problems, which is more often than participants in the matrix condition drew a matrix (27% of the problems; M = 4.28, SD = 5.08), $t(52) = 2.85$, $p < .01$. Perhaps the participants in the matrix condition were not fully comfortable with the use of a matrix, even after training; indeed, a few participants in the matrix condition used other types of representations to aid their problem solutions. As presented in the instructional materials, the matrix can be used to solve both joint- and total-probability problems, and we expected that the efficiency of learning one representation as opposed to two (i.e., tree and Venn diagrams) might be a more efficient approach. In future research we will attempt to use stronger scaffolding to facilitate the acquisition of the matrix as an approach to solving probability problems.

It may also be possible that requiring students to use such representations increases extraneous cognitive load rather than germane cognitive load (Sweller, 1988). The process of constructing the representation may be placing such a high demand on working memory that not enough capacity remains to conduct the work of utilizing the representation effectively. If this is the case, then we should expect that over time, with extended practice, the demand placed on working memory by the representations should be reduced, allowing more memory resources to be allocated to the efficient use of the representations. Future research should also address this issue.

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